2008 BC Form B/6 No Calculator

Let f be the function given by $f(x) = \frac{2x}{1+x^2}$

(a) Write the first four nonzero terms and the general term of the Taylor series for f about x=0.

(b) Does the series found in part (a), when evaluated at x=1, converge to $f(1)$? Explain why or why not.

(c) The derivative of $\ln(1+x^2)$ is $\frac{2x}{1+x^2}$. Write the first four nonzero terms of the Taylor series for $\ln(1+x^2)$ about x=0.

(d) Use the series found in part (c) to find a rational number A such that $\left|A - ln\left(\frac{5}{4}\right)\right| < \frac{1}{100}$. Justify your answer.

The function f has derivatives of all orders for all real numbers x. Assume $f(2) = -3$, $f'(2) = 5$, $f''(2) = 3$, and $f'''(2) = -8$.

(a) Write the third-degree Taylor polynomial for f about $x = 2$ and use it to approximate $f(1.5)$.

(b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \leq 3$ for all x in the closed interval $[1.5, 2]$. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \neq -5$.

(c) Write the fourth-degree Taylor polynomial, $P(x)$, for $g(x) = f\left(x^2 + 2\right)$ about $x = 0$. Use P to explain why g must have a relative minimum at $x = 0$.